Instanton Molecules in Finite Temperature QCD?

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Outline

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QCD at Finite Temperature

Under "normal" conditions we never observe free quarks: confinement.

Does this persist under high temperature/density conditions?

Or is there a phase transition at some finite temperature/chemical potential?

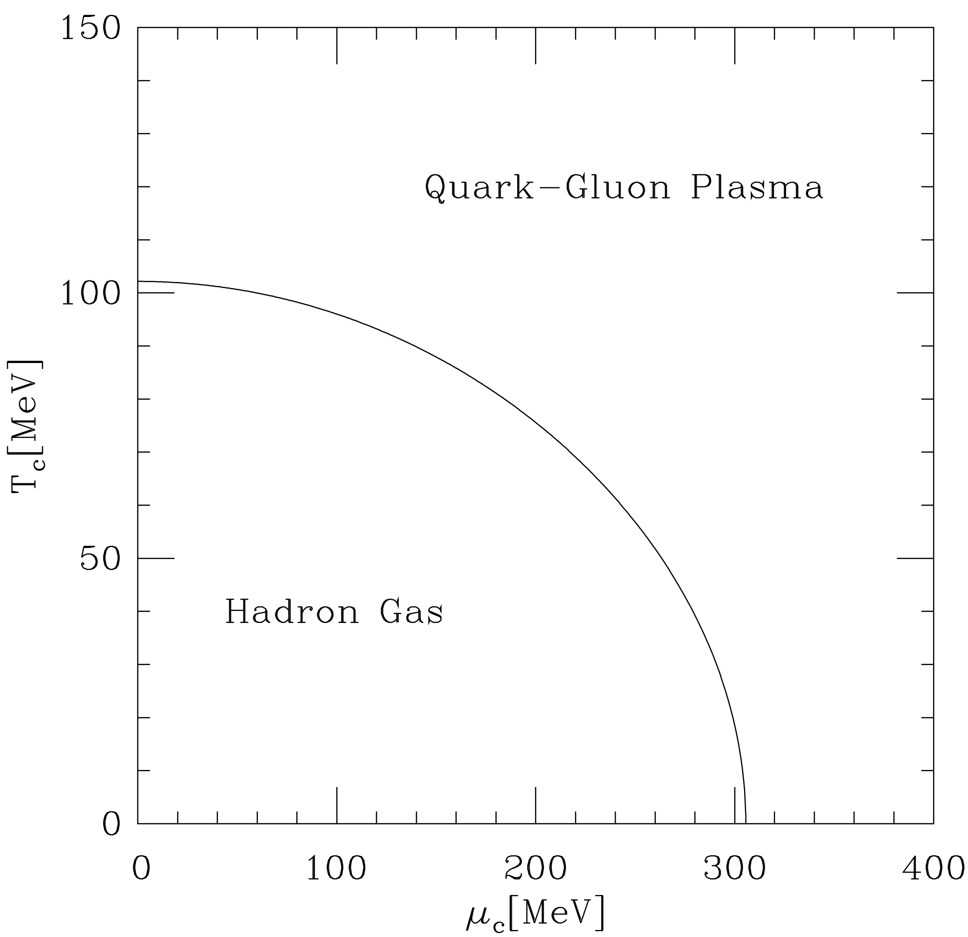
Since the 70's there have been speculations (Polyakov, Susskind) of a phase transition:

Low T	High T
Hadronic matter	Quark-gluon plasma
Confined quarks and gluons	Debye screening of color
Broken chiral symmetry	Restored chiral symmetry

RG arguments suggest that the QCD coupling decreases at high densities and temperatures (asymptotic freedom).

A naive thermodynamic analysis treating the quarks and gluons as free ultra-relativistic Fermi and Bose gases with a vacuum pressure B (MIT Bag model) gives the phase boundary ($N_f = 2$)

$$B = T_c^4 \left[\frac{37\pi^2}{90} + \left(\frac{\mu_c}{T_c}\right)^2 + \frac{1}{2\pi^2}\left(\frac{\mu_c}{T_c}\right)^4 \right]$$



Finite Temp. Field Theory

For a 4-dimensional QFT by analytical continuation we get

$$t \to -i\tau \quad \Rightarrow \quad e^{-iHt} \to e^{-H\tau}$$

so interpreting the Euclidean time as

$$\tau = \beta = \frac{1}{T}$$

allows us to connect the 4-d QFT to a 3-d statistical *equilibrium* system at finite temperature T.

On the lattice, the finite temperature condition is thus achieved by choosing a *finite periodic* time direction.

The partition function is

$$\mathcal{Z} = \mathrm{Tr}\ e^{-H/T} = \int \mathcal{D}\phi\ \langle \phi | e^{-H/T} | \phi \rangle$$

and expectation values are given by

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \mathrm{Tr}\ (\mathcal{O} e^{-H/T})$$

The matrix element in $\mathcal{Z}$ can be expressed as a path integral with periodic b.c's in time. This gives us, including both gauge and fermion fields

$$\mathcal{Z} = \int_{\text{per}} \mathcal{D}A_\mu \int_{\text{aper}} \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}}$$

The periodic b.c's for bosons give the discrete set of energy eigenvalues (Matsubara frequencies):

$$\omega_n = \frac{2\pi n}{T}, \;\; n = 0, 1, 2, \ldots$$

while for fermions we have the anti-periodic case with energies

$$\omega_n = \frac{(2n+1)\pi}{T}, \;\; n = 0, 1, 2, \ldots$$

Propagators at Finite T

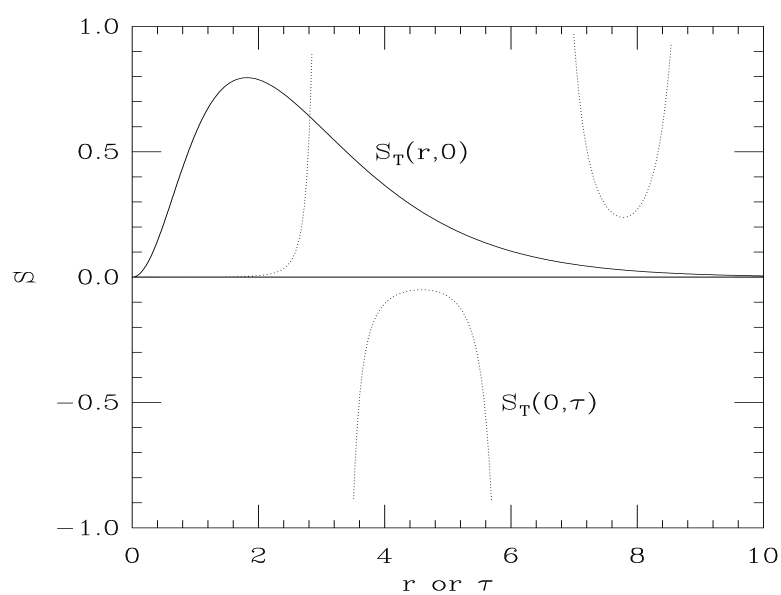
At finite T the free massless fermion propagator in the spatial direction is given by

$$S_T(r,0) = \frac{i\overrightarrow{\gamma}\cdot\overrightarrow{r}}{2\pi^2 r^4} z e^{-z} \left[\frac{(z+1)+(z-1)e^{-2z}}{\left(1+e^{-2z}\right)^2}\right]$$

where $z = \pi r T$, and in the τ direction by

$$S_T(0,\tau) = \frac{i\gamma_4}{2\pi^2\tau^3}\frac{y^3}{2}\frac{1+\cos^2(y)}{\sin^3(y)}$$

where $y = \pi\tau T$.



Instantons at $T = 0$

In QCD, chiral symmetry is broken even at $m_q = 0$. The *dynamical quark* topological susceptibility is

$$\chi_{\text{top}} = \left\langle \bar{\psi}\psi \right\rangle \frac{m_q}{N_f^2} + O(m_\pi^4)$$

and (Casher-Banks formula)

$$\left\langle \bar{\psi}\psi \right\rangle = \pi\rho(0),\ \rho(\lambda \sim 0) \sim \sqrt{\frac{n_I + n_A}{V}}.$$

where $\rho(\lambda)$ is the eigenmode density.

Thus,

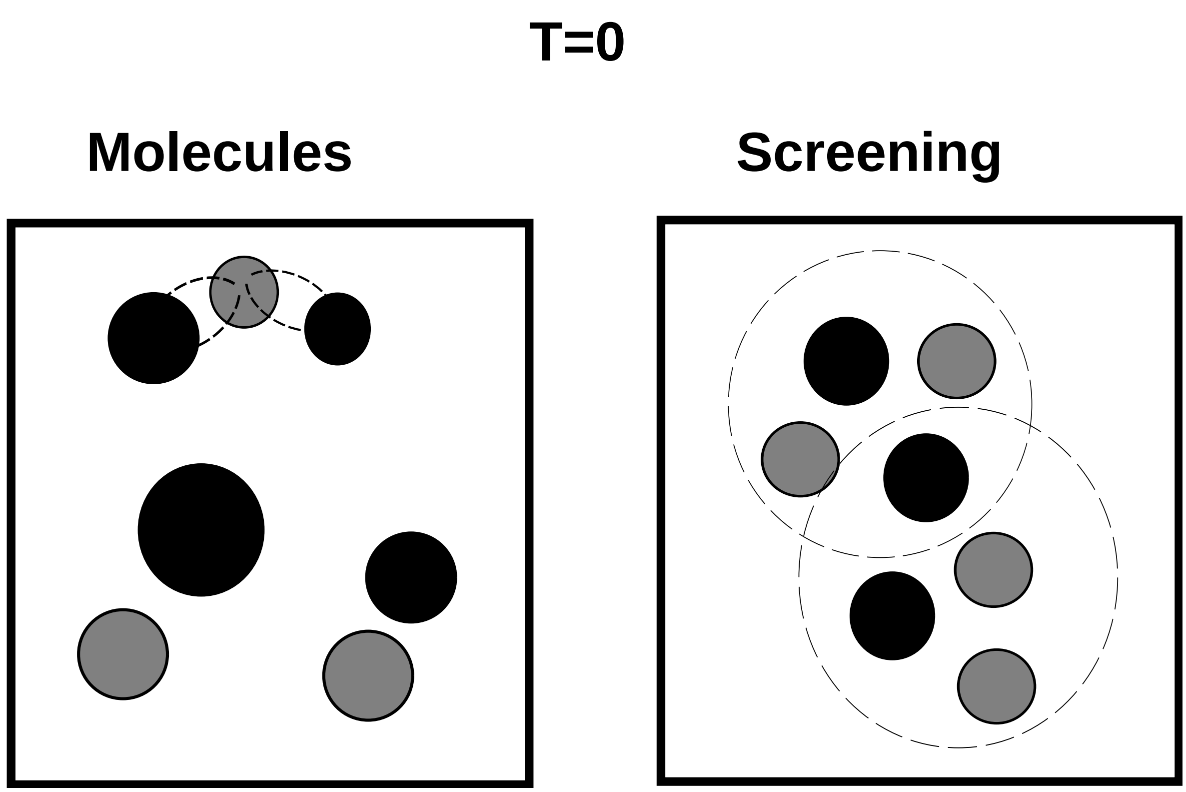
$$m_q \to 0 \quad \Rightarrow \quad \begin{cases} Q = n_I - n_A \to 0 \\ n_I + n_A \to \text{finite} \end{cases}$$

and the topological charge has to be screened.

Screening is due to the attractive interaction between I and A induced by the fermions.

Screening is possible by

1. tight molecule formation (could restore chiral symmetry but likely at finite T)
2. cloud of oppositely charged objects
3. the combination of molecules and clouds



Instantons at Finite T

We've seen the fermion propagators to be highly anisotropic, so we expect anisotropy effects in dynamical simulations.

But what about the instantons themselves?

The fermion determinant is calculated from the overlap matrix elements T_{IA} with the finite temperature zero modes (instanton related). We have

$$T_{IA} = u_4 f_1 + (\overrightarrow{u} \cdot \hat{r}) f_2$$

with the asymptotic forms for f_i being

$$\begin{aligned} f_1^{\text{as}} &= i\frac{\pi^2}{\beta} \sin\left(\frac{\pi\tau}{\beta}\right) \exp\left(-\frac{\pi r}{\beta}\right) \\ f_2^{\text{as}} &= i\frac{\pi^2}{\beta} \cos\left(\frac{\pi\tau}{\beta}\right) \exp\left(-\frac{\pi r}{\beta}\right) \end{aligned}$$

These forms strengthen the expectation that there should be strong correlations in the time direction and weak in the spatial directions.

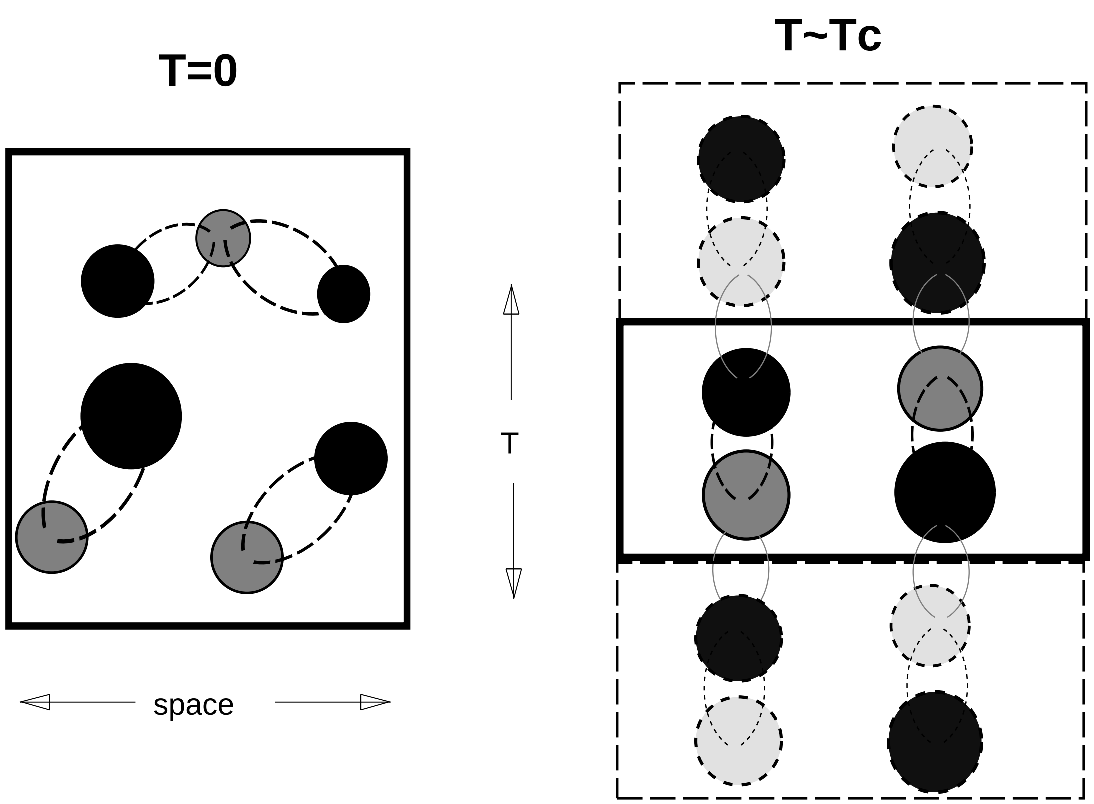
At $T \geq T_c$ chiral symmetry is restored:

$$\left\langle \bar{\psi}\psi \right\rangle = 0 \quad \Rightarrow \quad \rho(\lambda \sim 0) = 0$$

This can be explained in two ways:

- All instantons disappear
- The instantons form tight molecules, which induces large splittings in the zero mode spectrum.

The data seems to support the second option:



Data Setup

We measure the correlator

$$c(r,t) = \frac{1}{V}\int d^4x'\, q(r',t')q(r'+r,t'+t)$$

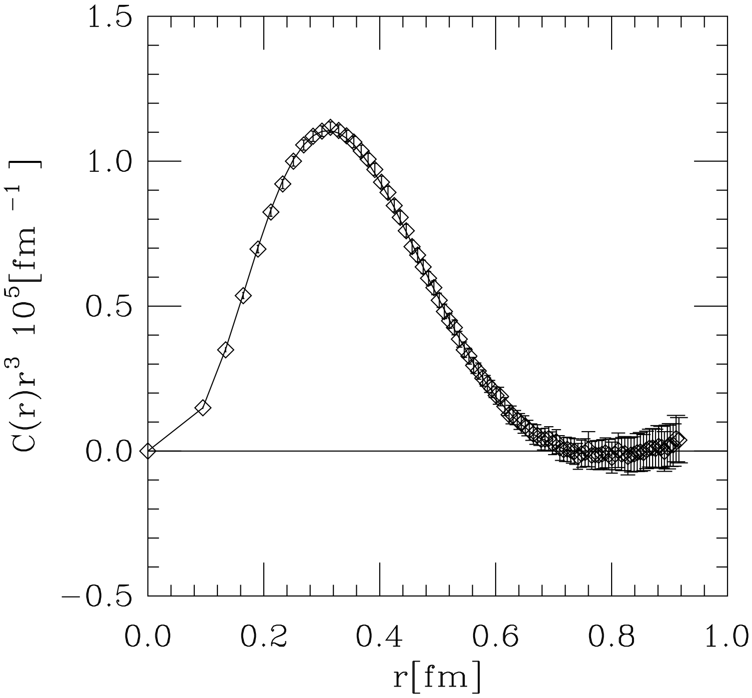
on MILC Kogut-Susskind dynamical lattices.

Lattices: $24^3 \times 12$, ~ 150 per β value.

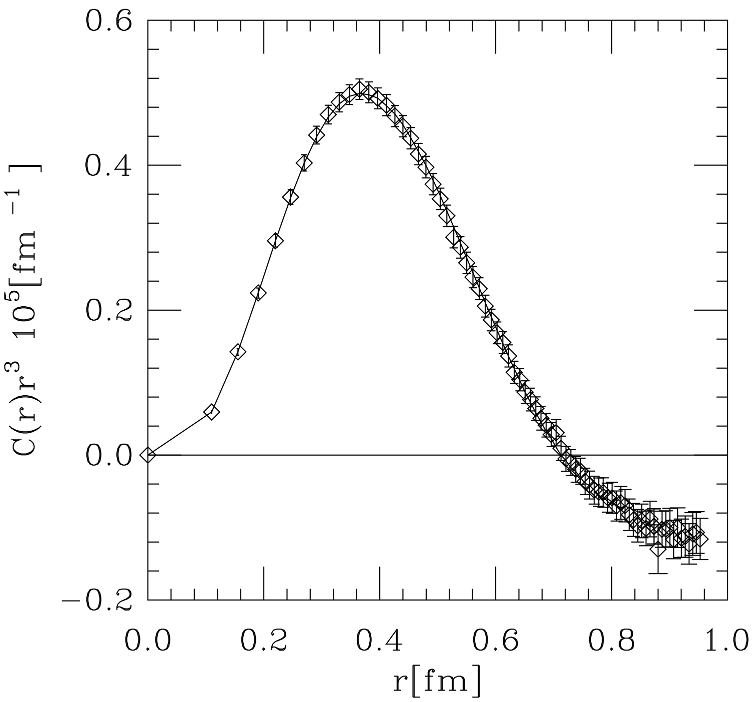
m	β	a (fm)	T_c
0.008	5.65	0.112	5.65 – 5.70
	5.725	0.099	
	5.85	0.093	
0.016	5.70	0.118	
	5.75	0.116	5.75 – 5.80
	5.85	0.111	

$T = 0$ **data**

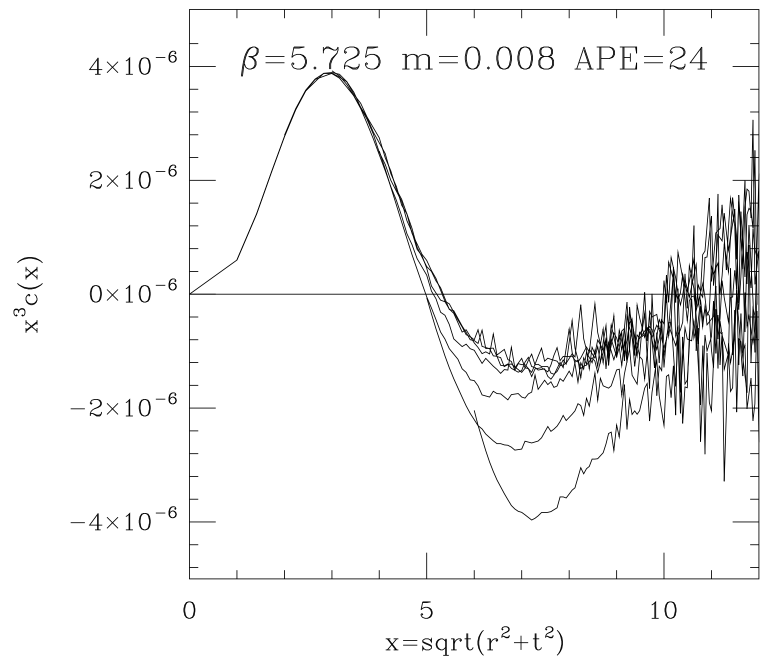
$C(r)r^3$, pure gauge configurations

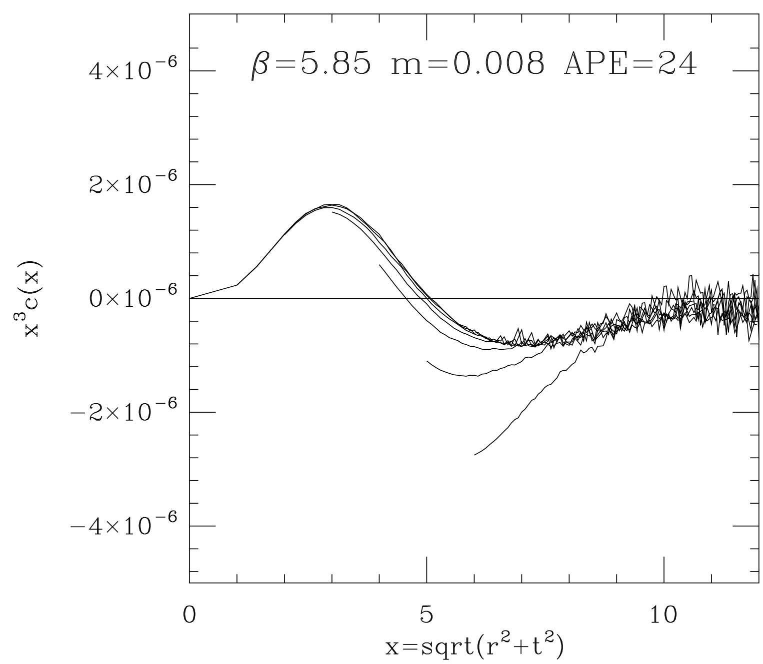


$C(r)r^3$, dynamical configurations



Time-separated data at finite T



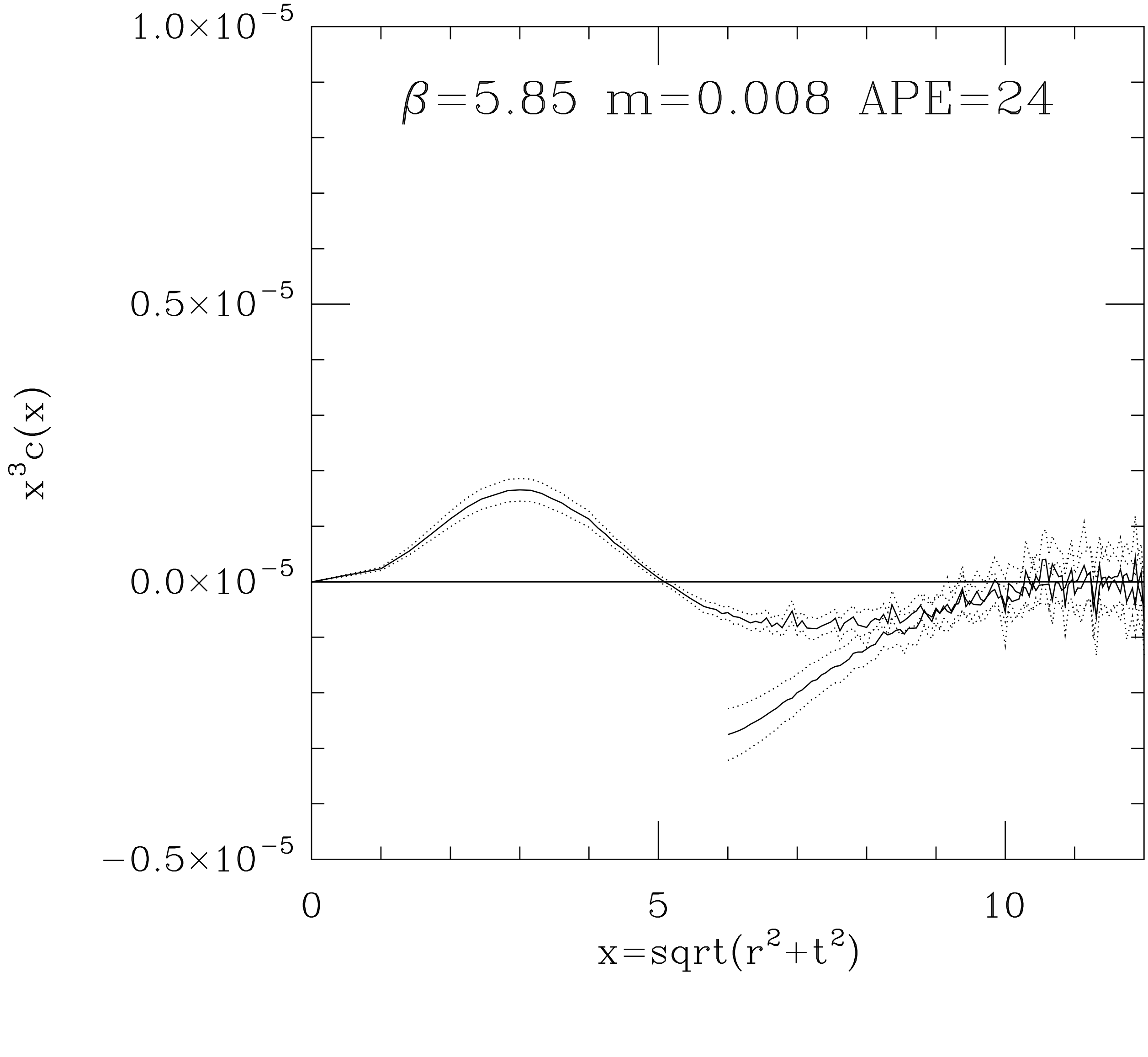


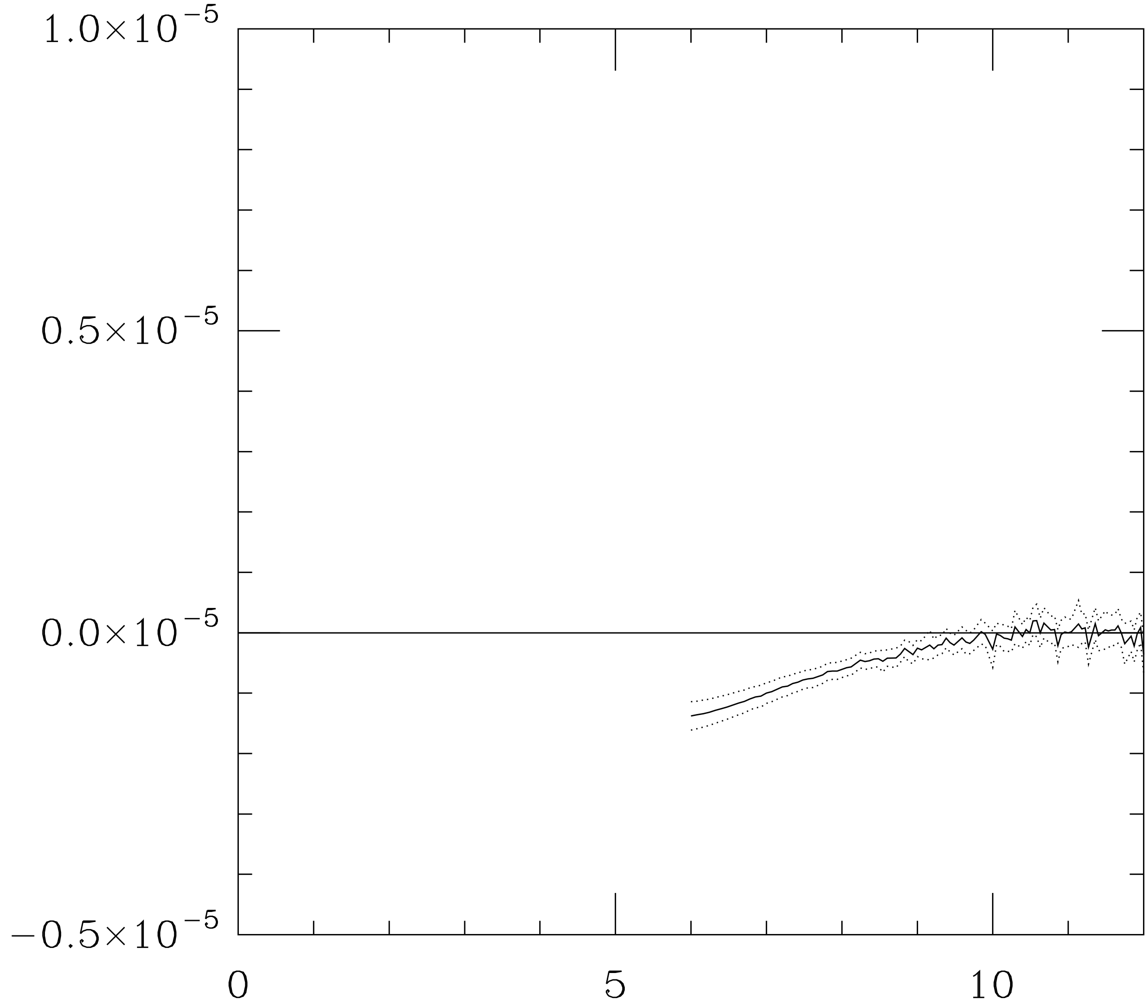
With $a \sim 0.1$ fm, we expect maximum pairing at

$$x \sim \frac{0.6}{a} = \left(\frac{2\bar{\rho}}{a}\right) \sim 6,$$

consistent with plot.

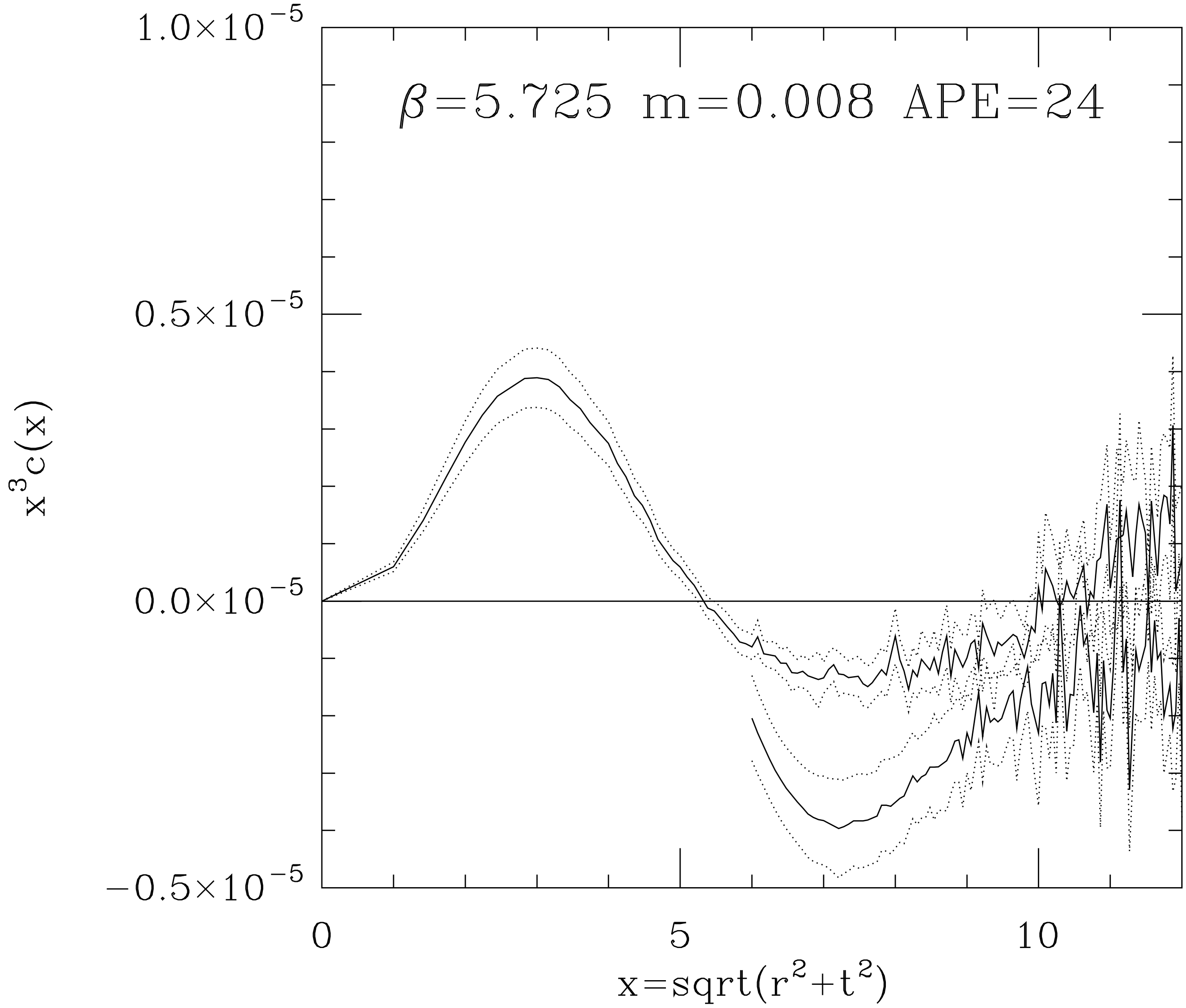
$T = 0$ and $T = 6$ (with errors)

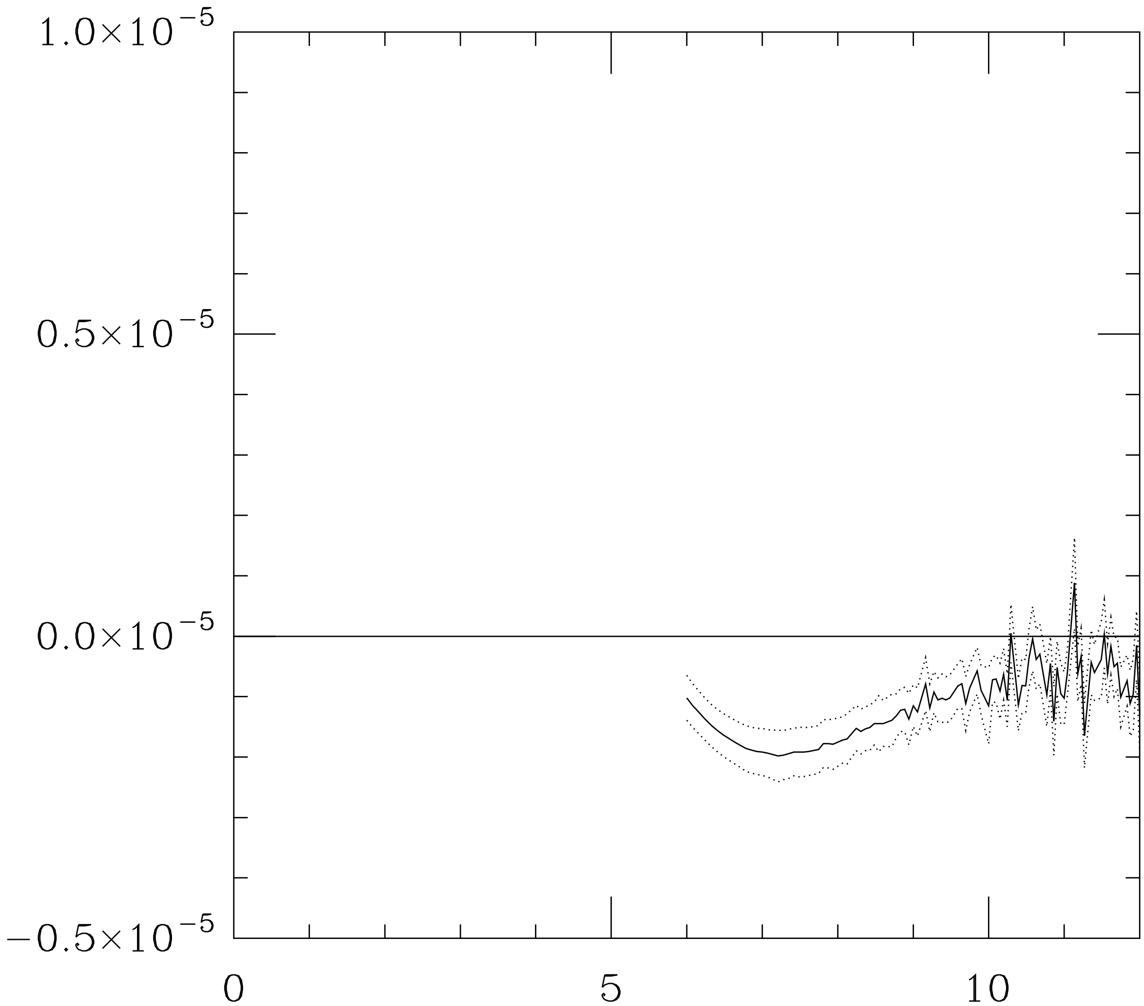




13-a

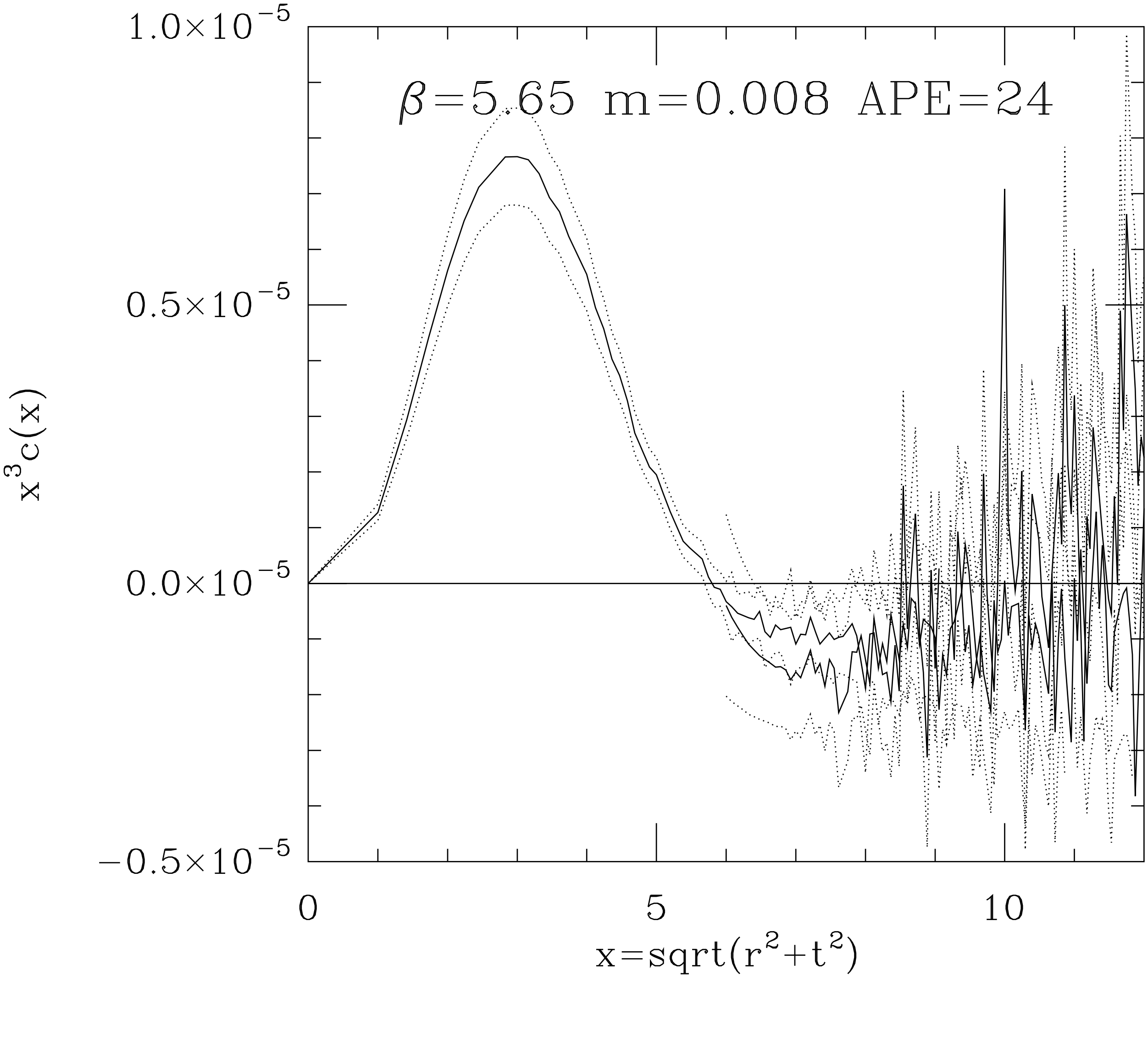
$T = 0$ and $T = 6$ (with errors)





1.0×10⁻⁵
0.5×10⁻⁵
0.0×10⁻⁵
−0.5×10⁻⁵
0
5
10

$T = 0$ and $T = 6$ (with errors)



Fitting

We first try to fit this correlation data to the simple model of N_0 instantons on the lattice with n_p of them paired:

$$c(r,t;\rho,d) = N_0 c_0(r,t;\rho) - n_p c_0^p(r,t;\rho,d)$$

where [(ρ,d) implied]

$$\begin{aligned} c_0^{(p)}(r,t) &= \frac{1}{V}\int d^4x'\, q_0^{(p)}(r',t')q_0^{(p)}(r'+r,t'+t) \\ q_0^p(r,t;\rho,d) &= \langle q_0(r,t;\rho) + \overline{q_0}(r+d,t+d;\rho)\rangle_{\text{angles}} \end{aligned}$$

and $q_0(r,t;\rho)$ is the single-instanton charge density

$$q_0 = (F^a_{\mu\nu})^2 = \frac{192\rho^4}{(x^2+\rho^2)^4}$$

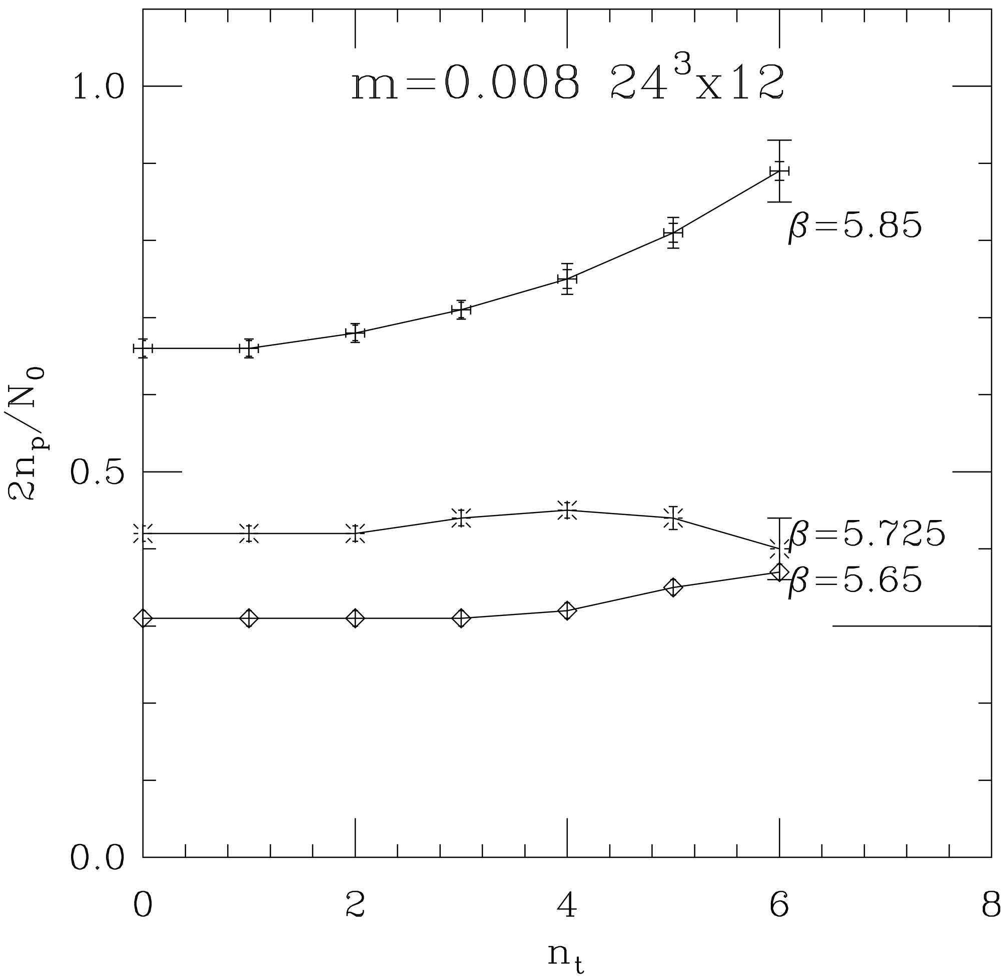
which corresponds to the BPST instanton solution

$$A^a_\mu(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2}$$

Fitting results

We fit the data with the parameters $N_0, n_p, d, \bar{\rho}$ with N_0 and $\bar{\rho}$ common for all time-slices, but d and $\bar{\rho}$ time-slice dependent.

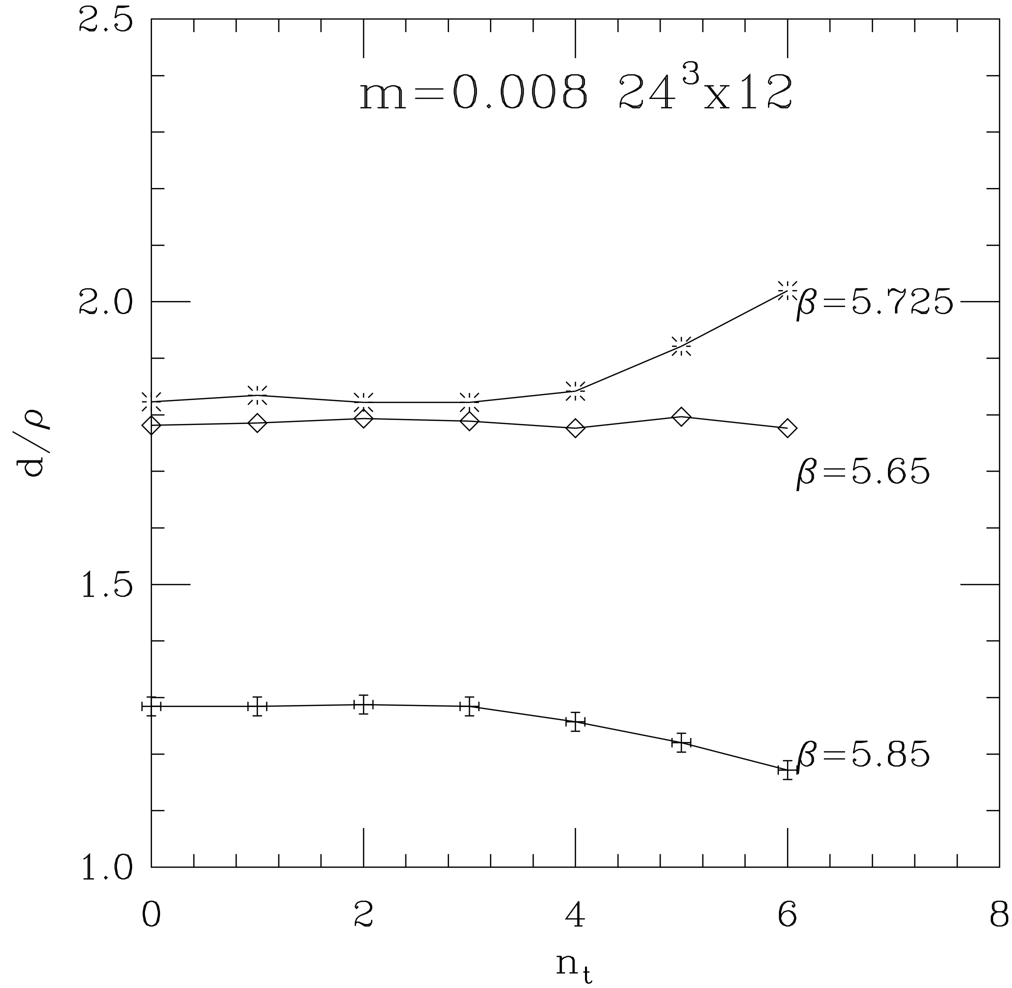
$2n_p/N_0$ as a function of t



The horizontal line shows the $T = 0$ result.

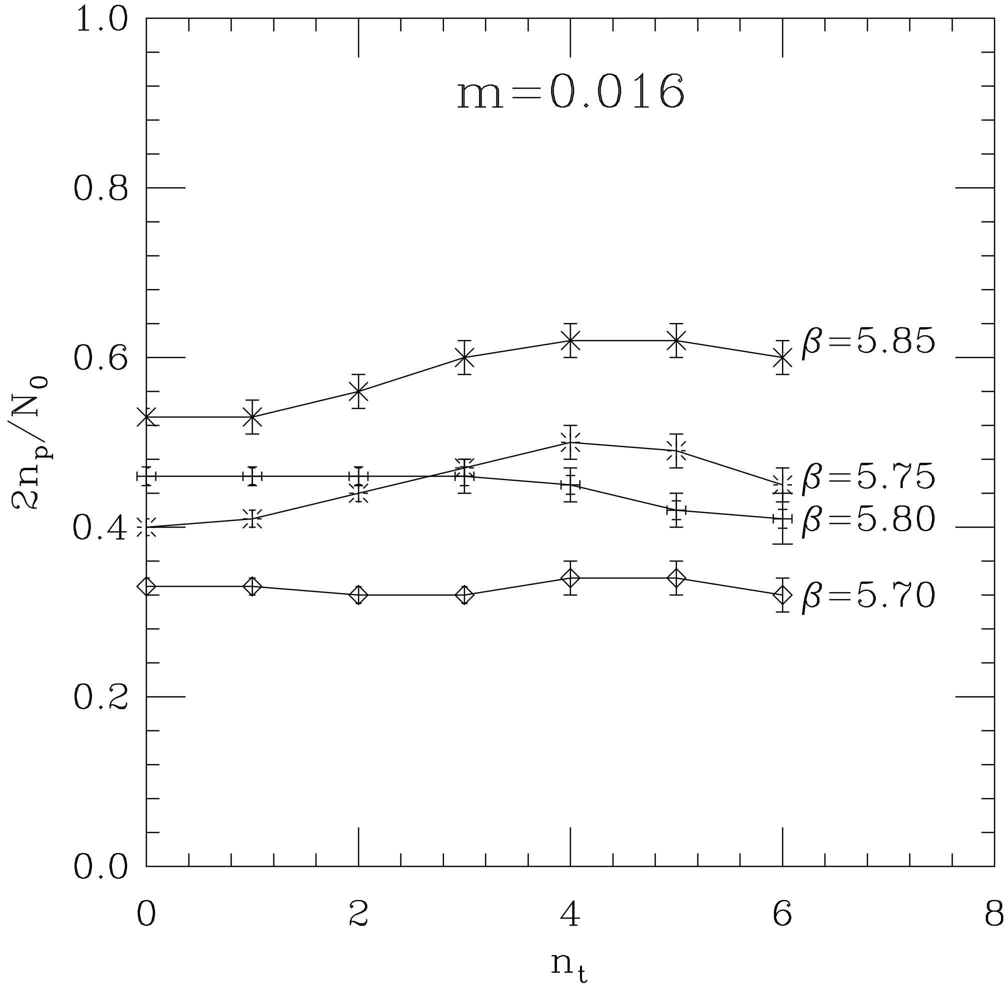
The fits have small χ^2 yet something strange happens to the separation of the pairs:

$d/\bar{\rho}$ as a function of t



The molecules pull together as $T \nearrow T_c$. At $\beta = 5.85$ they form very close pairs, "dumbbells".

$2n_p/N_0$ as a function of t



No signal for polarization - too heavy quark mass or $\beta = 5.85$ is too small?

Streamlines

In order to improve or model, we are looking for a more realistic I-A ansatz. The Yang-Mills action

$$S = \frac{1}{4}\int d^4x\, F^a_{\mu\nu}(x) F^a_{\mu\nu}(x)$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{abc} A^b_\mu A^c_\nu$$

is invariant under the conformal transformation

$$A^a_\mu \to \frac{1}{x^2}\left(\delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}\right) A^a_\nu\left(\frac{x_\alpha}{x^2}\right)$$

and the scale transformation

$$A^a_\mu \to \lambda A^a_\mu(\lambda x)\,.$$

Furthermore, a single-instanton solution can be transformed into an anti-instanton via a combined conformal-scale transformation.

We can then start with the naive form

$$A^a_\mu(x,\lambda) = 2\eta_{a\mu\nu} x_\nu \left[\frac{1}{x^2+\rho^2/\lambda} - \frac{1}{x^2+\rho^2\lambda}\right]$$

and obtain a reasonable ansatz for a separated IA pair, which will then be used for fitting.

At infinite separation this configuration is the starting point for the so called *streamline* solution.

Since the net topological charge is $Q = 0$, this configuration can be smoothly deformed to the trivial zero action vacuum.

The evolution of the infinitely separated pair is followed along lines perpendicular to the planes of constant action. This method provides a "best" ansatz for IA pairs at finite separation, but requires numerical solutions.

However, the form resulting after transformations from the above ansatz,

$$\begin{aligned}\frac{A^a_\mu}{2} \;=\;& \frac{\eta^a_{\mu\nu}(x_\nu - x_{1\nu})}{(x-x_1)^2+\rho_1^2} + \eta^a_{\mu\nu}\left[\frac{x_\nu}{x^2} - \frac{x_\nu - x_{2\nu}}{(x-x_2)^2}\right] \\ &+ \frac{\eta^a_{\mu\nu}(x_\nu - x_{2\nu})\rho_2^2}{(x-x_2)^2\left[(x-x_2)^2+\rho_2^2\right]}\end{aligned}$$

is very close to the numerical solution, and we will use this for fitting the correlation data.

This is still being worked on.

“Conclusions”, what next?

- We seem to see instanton molecules with an anisotropic distribution (polarization in the T direction).

- We need actual numbers to support the plot observations quantitatively.

- Complete analysis of full data set (binning needed to take care of correlations).

- Complete fitting work with the streamline form.

Questions - Anna:

- periodic/anti-periodic bc's?

- why $Q = 0$ if chiral symm is broken?

- why do we say that ch.symm is broken at $m_q = 0$ if

$$\chi_{top} \to 0 \quad as \quad m_q \to 0\,?$$